Fast and accurate boundary element methods

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We discuss efficient numerical methods for the boundary integral formulation of various three dimensional boundary value problems for the Laplace equation, Helmholtz equation and for the system of Lamé equations.
The corresponding boundary integral equations will be discretised using Galerkin method leading to a system of linear equations with a dense matrix A of some dimension N. A naive strategy for the solution of the corresponding linear systems would need at least $O(N^2)$ arithmetical operations and memory. Methods such as fast multipole [1] and panel clustering [2] provide an approximation to the solution vector x in almost linear complexity. These methods are based on explicitly given kernel approximations by degenerate kernels, i.e. a finite sum of separable functions, which may be seen as a blockwise low-rank approximation of the system matrix. The blockwise approximant permits a fast matrix-vector multiplication, which can be exploited in iterative solvers, and can be stored efficiently. In contrast to the methods mentioned, the Adaptive Cross Approximation method (ACA) [3] generates the low-rank approximant from the matrix itself using only few entries and without using any explicit a priori known degenerate kernel approximation. See also the recent monograph [4]. The efficiency and convergence properties of the numerical method (Galerkin discretisation, ACA approximation of matrices, iterative solution) will be illustrated for a number of different boundary value problems and for different surfaces.

References

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